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# TOPOLOGY AND DYNAMICS OF A FLOW THAT HAS A NON-SADDLE SET OR A $W$-SET

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Abstract. The aim of this paper is to study dynamical and topological properties of a flow in the region of influence of an isolated non-saddle set or a $W$-set in a manifold. These are certain classes of compact invariant sets in whose vicinity the asymptotic behaviour of the flow is somewhat controlled. We are mainly concerned with global properties of the dynamics and establish cohomological relations between the non-saddle set and the manifold. As a consequence we obtain a dynamical classification of surfaces (orientable and non-orientable). We also examine robustness and bifurcation properties of non-saddle-sets and study in detail the behavior of $W$-sets in 2-manifolds.

## 1. Introduction

In this paper we study topological aspects of the theory of non-saddle sets and $W$-sets and of their region of influence. These classes of compact invariant sets will be defined in next paragraphs. Their common feature is that the long term behaviour of the flow in their vicinity is controlled to a certain extent. Thus, in some respects, the study of this kind of sets can be seen as a general theory of stability and attraction, which extends the classical one and includes recent developments such as the theory of unstable attractors with no external explosions. We are mainly concerned with global properties of the dynamics.

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