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MOUNTAIN PASS SOLUTIONS FOR AN ENTIRE SEMIPOSITONE PROBLEM INVOLVING THE GRUSHIN SUBELLIPTIC OPERATOR

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Abstract. For $N \geq 3$ we study the following semipositone problem

$$-\Delta_\gamma u = g(z) f_a(u) \quad \text{in } \mathbb{R}^N,$$

where Δ_γ is the Grushin operator

$$\Delta_\gamma u(z) = \Delta_x u(z) + |x|^{2\gamma} \Delta_y u(z) \quad (\gamma \geq 0),$$

$z = (x, y) \in \mathbb{R}^N = \mathbb{R}^m \times \mathbb{R}^\ell$, $g \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ is a positive function, $a > 0$ is a parameter and f_a is a continuous function on $\mathbb{R}$ that coincides with $f(t) - a$ for $t \in \mathbb{R}^+$, where f is a continuous function with subcritical and Ambrosetti–Rabinowitz type growth and which satisfies $f(0) = 0$. Depending on the range of a, we obtain the existence of mountain pass solutions in a suitable Sobolev space associated to the Grushin operator, extending the results found in [5] to the Grushin operator.

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