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DYNAMICAL SYSTEM DESCRIBING CLOUD OF PARTICLES IN RELATIVISTIC AND NON-RELATIVISTIC FRAMEWORK

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Abstract. We consider a fairly general class of dynamical systems under assumptions that guarantee the existence of a Lyapunov function around a nontrivial stationary point. Moreover, we prove the existence of a heteroclinic trajectory. Finally, using geometric and topological reasoning, we establish an upper bound for this trajectory. The results can be interpreted as limit theorems in terms of integrated densities for astrophysical models in both relativistic and classical frameworks. These models encompass the static Smoluchowski–Poisson and Tolman–Oppenheimer-Volkoff equations.

1. Introduction

We consider the system

$$x' = y - x, \qquad y' = a(x)\, y - b(x)\, y^2,$$

where a and b are smooth functions defined on some interval. Assuming appropriate structural conditions on a and b, with $z > 0$ being a root of the right-hand side of the second equation (i.e. $a(z) = zb(z)$), we can prove that

$$V(x, y) = zB(x) - A(x) + y - z - z \log\left(\frac{y}{z}\right)$$

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