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ON THE DEGENERATE ARNOLD CONJECTURE ON $\mathbb{T}^{2M} \times \mathbb{CP}^N$

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Abstract. In the 1960s Arnold conjectured that a Hamiltonian diffeomorphism of a closed connected symplectic manifold (M, ω) should have at least as many contractible fixed points as a smooth function on M has critical points. Such a conjecture can be seen as a natural generalization of Poincaré's last geometric theorem and represents one of the most famous problems in symplectic geometry – still open today in its full generality. In this paper, we build on a recent approach of the authors and Izydorek to the Arnold conjecture on $\mathbb{CP}^n$ to show that the (degenerate) Arnold conjecture holds for Hamiltonian diffeomorphisms ϕ of $\mathbb{T}^{2m} \times \mathbb{CP}^n$, $m, n \geq 1$, which are C^0-close to the identity in the $\mathbb{CP}^n$-direction, namely that any such ϕ has at least $\mathrm{CL}\big(\mathbb{T}^{2m} \times \mathbb{CP}^n\big) + 1 = 2m + n + 1$ contractible fixed points.

1. Introduction

Given a closed symplectic manifold (M, ω) and a smooth time dependent one-periodic Hamiltonian function $H\colon \mathbb{T} \times M \to \mathbb{R}$, $\mathbb{T} := \mathbb{R}/\mathbb{Z}$, one defines the

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