Topological Methods in Nonlinear Analysis
Volume 66, No. 2, 2025, 815–840
DOI: 10.12775/TMNA.2025.053

PERIODIC SOLUTIONS FOR IMPULSIVE DIFFERENTIAL INCLUSIONS WITH STATE DEPENDENT IMPULSES

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Abstract. The paper investigates some qualitative properties of solutions to differential inclusions with state-dependent impulses. The first main objective is to prove that the mapping which assigns a set of solutions to the Cauchy problem for a given initial point is upper semicontinuous. This allows us to apply topological degree theory to the multivalued Poincaré operator along trajectories, which is the second main aim of the work, enabling us to establish the existence of periodic solutions. To verify the nonzero value of the topological degree, we utilize a generalized Krasnosel'skiĭ guiding function technique.

1. Introduction

In this paper, we study the behavior of trajectories of a *differential inclusion* $\dot{y}(t) \in F(t, y(t))$ on an interval $[0, T]$. Multivalued right-hand sides $F\colon [0,T] \times E \multimap E$ arise in many problems, such as those involving trajectories with velocities constrained to generalized gradients of nondifferentiable functions or in control problems where $F(t, y(t)) := \{f(t, y(t), u(t)) \mid u(t) \in U(y(t))\}$, with f being a continuous map and U describing the set of possible controls. *Impulsive differential problems*, first investigated by Milman and Myshkis [16], refer to differential equations (or inclusions) with dynamics that allow instantaneous jumps

2020 *Mathematics Subject Classification.* Primary: 34A60; Secondary: 34A37, 47H04, 34C25.

Key words and phrases. Differential inclusion; state-dependent impulses; periodic solution; guiding function; Poincaré translation operator; solution map.