

**POSITIVE GROUND STATE SOLUTIONS
OF THE CRITICAL NONLINEAR SCHRÖDINGER SYSTEM
WITH THE HARMONIC POTENTIALS
FOR THE COOPERATIVE CASE**

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ABSTRACT. In this paper, we study the following two coupled nonlinear Schrödinger system with critical exponent:

$$\begin{cases} -\Delta u + |x - x_0|^2 u - \lambda_1 u = \mu_1 u^{2^*-1} + \beta u^{2^*/2-1} v^{2^*/2}, & x \in \mathbb{R}^N, \\ -\Delta v + |x - y_0|^2 v - \lambda_2 v = \mu_2 v^{2^*-1} + \beta v^{2^*/2-1} u^{2^*/2}, & x \in \mathbb{R}^N, \\ u > 0, \quad v > 0, & x \in \mathbb{R}^N, \\ u(x) \rightarrow 0, \quad v(x) \rightarrow 0, & \text{as } |x| \rightarrow +\infty, \end{cases}$$

where $N \geq 3$, $2^* = 2N/(N-2)$, $\mu_1, \mu_2, \beta > 0$, $x_0, y_0 \in \mathbb{R}^N$ and $\lambda_1, \lambda_2 \in (\lambda_*, N)$ with $\lambda_* = 1$ for $N = 3$ and $\lambda_* = 0$ for $N \geq 4$. By the variational method, we show that the existence of a positive ground state solution for the above system under the cooperative case. Furthermore, we also partially reveal the influence mechanisms of x_0, y_0, λ_i and β on the existence of positive ground state solutions.

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