

LOWER BOUNDS FOR ADMISSIBLE VALUES OF THE TRAVELLING WAVE SPEED IN ASYMMETRICALLY SUPPORTED BEAM

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ABSTRACT. We study the admissible values of the wave speed c for which the beam equation with jumping nonlinearity possesses a travelling wave solution. In contrast to previously studied problems modelling suspension bridges, the presence of the term with negative part of the solution in the equation results in restrictions of c . In this paper, we provide the maximal wave speed range for which the existence of the travelling wave solution can be proved using the Mountain Pass Theorem. We also introduce its close connection with related Dirichlet problems and their Fučík spectra. Moreover, we present several analytical approximations of the main existence result with assumptions that are easy to verify. Finally, we formulate a conjecture that the infimum of the admissible wave speed range can be described by the Fučík spectrum of a simple periodic problem.

1. Motivation

When studying the existence of homoclinic travelling waves in asymmetrically supported vibrating beams, i.e., solutions of the problem

$$(1.1) \quad \begin{cases} u_{tt} + u_{xxxx} + au^+ - bu^- = 1, & x \in \mathbb{R}, t > 0, \\ u(x, t) \rightarrow 1/a, u_x(x, t) \rightarrow 0, & \text{for } |x| \rightarrow +\infty, \end{cases}$$

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Key words and phrases. Beam equation; jumping nonlinearity; travelling wave; Mountain Pass Theorem; Fučík spectrum; Swift–Hohenberg operator; Padé approximation.

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