

**CONSTRUCTION OF SOLUTIONS
TO A NONLINEAR CRITICAL ELLIPTIC SYSTEM
VIA LOCAL POHOZAEV IDENTITIES**

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ABSTRACT. In this paper, we investigate the following elliptic system with Sobolev critical growth

$$\begin{cases} -\Delta u + P(|y'|, y'')u = u^{2^*-1} + \frac{\beta}{2} u^{2^*/2-1} v^{2^*/2}, & y \in \mathbb{R}^N, \\ -\Delta v + Q(|y'|, y'')v = v^{2^*-1} + \frac{\beta}{2} v^{2^*/2-1} u^{2^*/2}, & y \in \mathbb{R}^N, \\ u, v > 0, & u, v \in H^1(\mathbb{R}^N), \end{cases}$$

where $(y', y'') \in \mathbb{R}^2 \times \mathbb{R}^{N-2}$, $P(|y'|, y''), Q(|y'|, y'')$ are bounded non-negative function in $\mathbb{R}^+ \times \mathbb{R}^{N-2}$, $2^* = 2N/(N-2)$. By combining a finite reduction argument and local Pohozaev type of identities, assuming that $N \geq 5$ and $r^2(P(r, y'') + \kappa^2 Q(r, y''))$ has a common topologically nontrivial critical point, we construct an unbounded sequence of non-radial positive vector solutions of synchronized type, whose energy can be made arbitrarily large. Our result extends the result of a single critical problem by Chen, Pistoia and Vaira (Discrete Contin. Dyn. Syst. **43** (2023), 482–506). The novelties mainly include the following two aspects. On one hand, when $N \geq 5$, the coupling exponent $2/(N-2) < 1$, which creates a great trouble for us to apply the perturbation argument directly. This constitutes the main difficulty different between the coupling system and a single equation. On the other hand, the weaker symmetry conditions of $P(y)$ and $Q(y)$ make us not estimate directly the corresponding derivatives of the reduced functional in locating the concentration points of the solutions, we employ some local Pohozaev identities to locate them.

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Key words and phrases. Critical elliptic systems; local Pohozaev identities; Lyapunov–Schmidt reduction; synchronized vector solutions.

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