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ANALYZING MULTIFILTERING FUNCTIONS USING MULTIPARAMETER DISCRETE MORSE THEORY

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Abstract. A multiparameter filtration, or a multifiltration, may in many cases be seen as the collection of sublevel sets of a vector function, which we call a multifiltering function. The main objective of this paper is to obtain a better understanding of such functions through multiparameter discrete Morse (`mdm`) theory, which is an extension of Morse-Forman theory to vector-valued functions. Notably, we prove algorithmically that any multifiltering function defined on a simplicial complex can always be approximated by a compatible `mdm` function. Moreover, we define the Pareto set of a discrete multifiltering function and show that the concept links directly to that of critical simplices of a `mdm` function. Finally, we experiment with these notions using triangular meshes.

1. Introduction

Topological data analysis (TDA) is a fast-growing branch of mathematics which proposes a panoply of tools to better understand and visualize data [18], [59], [68]. One of the most popular method in TDA is persistent homology,

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Key words and phrases. Discrete Morse theory; multiparameter persistent homology; multifiltering functions; discrete gradient field; Pareto set; critical components; topological data analysis.

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