

THE KERNEL SPACE OF LINEAR OPERATOR FOR A CLASS OF GRUSHIN EQUATION

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ABSTRACT. In this paper, we consider the kernel of linear operator for a class of Grushin equation. First, we study the kernel space of linear operator for a general Grushin equation. Then, we provide an exact expression for the kernel space of linear operator associated with a special Grushin equation. Finally, we prove the linear operator related to the singularly perturbed Grushin equation is invertible when restricted to the complement of its approximate kernel space.

1. Introduction

In this paper, we consider the following problem

$$(1.1) \quad -\Delta_\gamma u = f(u), \quad u > 0, \quad \text{in } \mathbb{R}^{N+l},$$

where Δ_γ is the well-known *Grushin operator* given by

$$(1.2) \quad \Delta_\gamma u(z) = \Delta_x u(z) + |x|^{2\gamma} \Delta_y u(z).$$

Δ_x and Δ_y are the Laplace operators in the variable x and y , respectively, with $z = (x, y) \in \mathbb{R}^N \times \mathbb{R}^l = \mathbb{R}^{N+l}$ and $N > 1$, $l > 1$, $N + l \geq 3$. Here, $\gamma \geq 0$ is a real number and

$$(1.3) \quad N_\gamma = N + (1 + \gamma)l$$

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