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RELATIVE SECTIONAL NUMBER AND THE COINCIDENCE PROPERTY

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Abstract. For a Hausdorff space Y, a topological space X and a map $g\colon X \to Y$, we present a connection between the relative sectional number of the first coordinate projection $\pi^Y_{2,1}\colon F(Y,2) \to Y$ with respect to g, and the coincidence property (CP) for $(X, Y; g)$, where $F(Y,2)$ stands for the ordered configuration space of 2 distinct points on Y, and $(X, Y; g)$ has the coincidence property CP if, for every map $f\colon X \to Y$, there is a point x of X such that $f(x) = g(x)$. Explicitly, we demonstrate that $(X, Y; g)$ has the CP if and only if 2 is the minimal cardinality of open covers $\{U_i\}_{1\leq i\leq n}$ of X such that each U_i admits a local lifting for g with respect to $\pi^Y_{2,1}$. This characterization connects a standard problem in coincidence theory to current research trends in sectional category and topological robotics. Motivated by this connection, we introduce the notion of relative topological complexity of a map.

1. Introduction, outline and main results

In this article "space" means a topological space, and by a "map" we will always mean a continuous map. Also by "fibration" we will mean Hurewicz fibration. We write $\overline{y_0}$ to denote the constant map in y_0.

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