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IMPROVED BOUNDS FOR FRACTIONAL INTEGRALS IN GENERALIZED LOCALLY CONVEX SPACES AND APPLICATIONS

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Abstract. Using Bielecki's idea, we begin by introducing generalized locally convex structures on n-Cartesian product of the set of continuous functions defined on the half-axis. Within this frame, we prove new boundedness results for generalized proportional fractional (GPF) integral operators of vector order involving maxima and deviating arguments. As a consequence, one of the well-known boundedness results for scalar Riemann–Liouville fractional integral operators is generalized and improved. As an application, a vector approach for coupled systems of nonlinear (GPF) differential equations with maxima is adopted. Based on our findings related to boundedness and using Perov's type fixed point theorem, we establish global existence-uniqueness results under less restrictive conditions compared to those commonly imposed in the literature.

1. Introduction

Let X be the set of all real continuous functions defined on the half-axis, n be a positive integer and for $\lambda > 0$ and $\mu > 1$, let H_μ^λ be a weight function defined appropriately via a continuous function H. Using Bielecki's idea, we begin by introducing the locally convex space (lcs) $X_{H_\mu^\lambda}$ made up of functions in X equipped with a family of weighted semi-norms $\left\{\|\cdot\|_{X_{H_\mu^\lambda},k}\right\}_{k\in\mathcal{K}}$ generating

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