Topological **M**ethods in **N**onlinear **A**nalysis
Volume 66, No. 1, 2025, 337–386
DOI: 10.12775/TMNA.2024.059

MINIMIZING TRAVELLING WAVES FOR THE ONE-DIMENSIONAL NONLINEAR SCHRÖDINGER EQUATIONS WITH NON-ZERO CONDITION AT INFINITY

Jordan Berthoumieu

Abstract. This paper deals with the existence of travelling wave solutions for a general one-dimensional nonlinear Schrödinger equation. We construct these solutions by minimizing the energy under the constraint of fixed momentum. We also prove that the family of minimizers is stable. Our method is based on recent articles about the orbital stability for the classical and nonlocal Gross–Pitaevskiĭ equations [4], [17]. It relies on a concentration-compactness theorem, which provides some compactness for the minimizing sequences and thus the convergence (up to a subsequence) towards a travelling wave solution.

1. Introduction

We are interested in the defocusing nonlinear Schrödinger equation

$$i\partial_t \Psi + \Delta \Psi + \Psi f(|\Psi|^2) = 0 \quad \text{on } \mathbb{R} \times \mathbb{R}. \tag{NLS}$$

This equation appears as a relevant model in condensed matter physics. In particular, it is relevant in the context of the Bose–Einstein condensation or

2020 *Mathematics Subject Classification.* Primary: 54C40, 14E20; Secondary: 46E25, 20C20.

Key words and phrases. Schrödinger equation; travelling waves; variational argument; orbital stability.

This work was supported by the CY Initiative of Excellence (Grant "Investissements d'Avenir" ANR-16-IDEX-0008).