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PROPER TOPOLOGICAL COMPLEXITY

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Abstract. We introduce and study the proper topological complexity of a given configuration space, a version of the classical invariant for which we require that the algorithm controlling the motion is able to avoid any possible choice of "unsafe" area. To make it a homotopy functorial invariant we characterize it as a particular instance of the exterior sectional category of an exterior map, an invariant of the exterior homotopy category which is also deeply analyzed.

1. Introduction

In broad terms, the topological complexity $\mathrm{TC}(X)$ of a motion on a given configuration space X, as originally introduced in [9], provides a lower bound of the number of "continuous instructions" of any algorithm controlling the given motion. One can also regard this invariant as the minimum number of navigational instabilities of the given motion.

However, to improve the efficiency of a given motion planner, one often needs to impose extra conditions. Accordingly, different versions of topological complexity, sharpening the original one, have appeared in recent years. For instance, for the *symmetric topological complexity* [11], the motion from a point of the configuration space to itself is required to be the constant path and the trajectories

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