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EXISTENCE AND UNIQUENESS OF A RENORMALIZED SOLUTION TO FRACTIONAL ELLIPTIC INCLUSIONS WITH L^1-DATA

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Abstract. Let $d \in \mathbb{N}$ and $\Omega \subset \mathbb{R}^d$ be bounded with Lipschitz boundary. We prove the existence and uniqueness of a renormalized solution to the inclusion problem

$$\begin{cases} \beta(u) + Lu \ni f & \text{in } \Omega, \\ u = 0 & \text{in } \partial\Omega, \end{cases}$$

where $f \in L^1(\Omega)$, the operator L generalizes the regional fractional p-Laplacian with $1 < p < \infty$ and β is a maximal monotone operator on $\mathbb{R}$. A comparison between renormalized and weak solutions is also discussed.

1. Introduction

Renormalized solutions were first introduced by DiPerna and Lions in [10]. The main idea is to tackle the problem of uniqueness for partial differential equations with L^1-data, which need not hold when distributional solutions are considered (cf. [18] and [17]). Since [10], various extensive studies have been

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