Topological **M**ethods in **N**onlinear **A**nalysis
Volume 66, No. 1, 2025, 215–235
DOI: 10.12775/TMNA.2025.008

INFINITELY MANY SMALL BOUNCING SOLUTIONS OF HILL'S TYPE IMPACT OSCILLATORS NEARBY THE ORIGIN

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Abstract. In this paper, we consider a class of Hill's type impact oscillators with super-linear restoring force nearby the origin. Infinitely many small and subharmonic bouncing solutions are obtained as well as symmetric subharmonic bouncing solutions of symmetric equations. The results are mainly obtained by using phase plane analysis, a generalized Poincaré–Birkhoff twist theorem and limiting arguments.

1. Introduction

Consider the nonlinear Hill's type impact oscillators with an obstacle

$$\begin{cases} x'' + q(t)g(x) + m(t,x) = 0, \quad x(t) > 0, \\ x(t) \geq 0, \\ \text{if } x(t_0) = 0, \text{ then } x'(t_0+) = -x'(t_0-), \end{cases} \tag{1.1}$$

where $q\colon [0,2\pi] \to \mathbb{R}^+$ is a continuous and 2π-periodic positive function, $g\colon \mathbb{R}_0^+ \to \mathbb{R}$ is locally Lipschitzian continuous satisfying $g(0) = 0$ and

(g_0) $g(x) > 0$, if $x > 0$,

(g_1) $\lim\limits_{x\to 0^+} \dfrac{g(x)}{x} = 0$,

2020 *Mathematics Subject Classification.* 34A36, 34C15, 34C25.

Key words and phrases. Nonlinear Hill equations; impact oscillators; periodic solutions; generalized Poincaré–Birkhoff twist theorem.

This work is supported by the NNSF of China (No. 12071410, 12071232, 12271119).