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CHARACTERIZATIONS OF DISTALITY VIA WEAK EQUICONTINUITY ON COMPACT HAUSDORFF SPACES

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Abstract. For an infinite discrete group G acting on a compact Hausdorff space X, we characterize distality via weak equicontinuity introduced by Li and Yang. In other words, we show that if a minimal system (X, G) admits an invariant measure then (X, G) is distal if and only if it is pairwise IP*-equicontinuous; if the product system $(X \times X, G)$ of a minimal system (X, G) has a dense set of minimal points and G is countable, then (X, G) is distal if and only if it is pairwise IP*-equicontinuous if and only if it is pairwise central*-equicontinuous. This is a generalization of compact metric space of Li and Yang (Discrete Contin. Dyn. Syst. **44** (2024), no. 1, 61–77, DOI: 10.3934/dcds.2023096).

1. Introduction

By a *topological dynamical system* or a *G-action* we mean a pair (X, G), where X is a compact Hausdorff space with the unique symmetric uniform structure $\mathcal{U}_X$ and G is an infinite discrete group acting continuously on X. When $G = \mathbb{Z}$ the action can be generated by a homeomorphism T from X to itself, in this case we will simply write the system as (X, T).

We say that a dynamical system (X, G) is *equicontinous* if, for any $\alpha \in \mathcal{U}_X$, there is a $\beta \in \mathcal{U}_X$ such that whenever $(x, y) \in \beta$, $(gx, gy) \in \alpha$, for all $g \in G$, and *distal* if for any two distinct points $x, y \in X$, there is an $\alpha \in \mathcal{U}_X$

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