Topological **M**ethods in **N**onlinear **A**nalysis
Volume 66, No. 1, 2025, 109–127
DOI: 10.12775/TMNA.2025.005

EXISTENCE OF GROUND STATE SOLUTIONS FOR THE NONLINEAR ELLIPTIC EQUATIONS WITH ZERO MASS ON LATTICE GRAPHS

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Abstract. In this paper, we will study the following nonlinear elliptic equation with zero mass on the lattice graph

$$\text{(A)} \qquad \begin{cases} -\Delta_p u = K(x)f(u) & \text{in } \mathbb{Z}^N, \\ u \in D^{1,p}(\mathbb{Z}^N), \end{cases}$$

where $N \geq 3$, $1 < p < N$, K is a nonnegative potential function, f is a continuous function with quasicritical growth or supercritical growth. By employing variational methods, we establish the existence of ground states for the above equation with an asymptotically periodic potential and vanishing potential at infinity. For the case of asymptotically periodic potential, we also generalize the main result from $\mathbb{Z}^N$ to quasi-transitive graphs.

1. Introduction

The following semilinear elliptic problems have been extensively studied

$$(1.1) \qquad -\Delta u = K(x)f(u), \quad x \in \mathbb{R}^N,$$

2020 *Mathematics Subject Classification.* Primary: 35R02; Secondary: 35J60, 58E30.
Key words and phrases. Lattice graphs; zero mass; ground states; variational methods; elliptic equations.
The first author was partially supported by National Natural Science Foundation of China (No. 12171152) and FAPESP/Brazil grant # 2022/09273-9. This work was carried out when C. Ji visited Universidade Federal de São Carlos. He is very grateful to the members of Departamento de Matemática for their warm hospitality.
The second author was supported by FAPESP/Brazil grant # 2022/16407-1 and CNPq/Brazil grant #303256/2022-2.