Topological **M**ethods in **N**onlinear **A**nalysis
Volume 66, No. 1, 2025, 91–107
DOI: 10.12775/TMNA.2025.003

© 2025 Juliusz Schauder Centre for Nonlinear Studies
Nicolaus Copernicus University in Toruń

DIRECTED TOPOLOGICAL COMPLEXITY OF POSET-STRATIFIED SPACES

Kohei Tanaka

Abstract. This paper studies directed motion planning algorithms on a CW complex, or more generally, a space decomposed by a poset. We demonstrate that a regular CW complex admits a global directed motion planning algorithm that is compatible with the face poset. Furthermore, as a non-regular case, we compute the directed topological complexity of the directed circle associated with the minimal cell decomposition.

1. Introduction

The topological complexity (TC) of a space is a numerical homotopy invariant related to the robot motion planning problem. For a path-connected space X, a motion planning algorithm (or motion planner) on a subspace $U \subset X \times X$ is a continuous local strict section $U \to X^I = \mathrm{Map}([0,1], X)$ of the free path-fibration $X^I \to X \times X$. The topological complexity $\mathrm{TC}(X)$ is defined as one less than the smallest size of (open) subspaces covering $X \times X$ with motion planning algorithms. Since its introduction by Farber [5], TC has been studied and developed under various practical conditions.

This study focuses on the directed topological complexity $\overrightarrow{\mathrm{TC}}$ of a d-space (directed space) [9] related to the concurrency theory. A d-space X is equipped

2020 *Mathematics Subject Classification.* Primary: 55M30; Secondary: 05E45, 55U05.
Key words and phrases. Directed topological complexity; directed motion planning; poset-stratified space.
This work was partially supported by JSPS KAKENHI Grant Number JP20K03607.