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FAN-HEMICONTINUITY FOR THE GRADIENT OF THE NORM IN SEVERAL REFLEXIVE BANACH SPACES

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Abstract. The present study approaches variational inequalities governed by Fan-hemicontinuous operators. The Fan-hemicontinuity was recently proved ([5]) for the gradient of the norm defined on its scalarly-positive, closed convex subdomain of a Hilbert space. The aim of the present study is to establish whether the positive result on Fan-hemicontinuity can be extended to an arbitrary reflexive Banach space. In this matter it is natural to consider the duality map and its topological properties. Based on the weak-weak sequential continuity of the generalized duality map, the discussed property holds on ℓ^p, $1 < p < +\infty$ and does not hold on L^p, $p \neq 2$, thus restricting the space setting where weak compactness results are applicable.

1. Introduction

It is well-known that the class of pseudomonotone operators is invariant under addition. In particular, it is the case for the operator generated by the energy functional of the mathematical pendulum sumated with $\nabla\|\cdot\|$ and/or $\nabla(-1/\|\cdot\|^q)$, $q \geq 1$ (for $q = 1$ is the Newtonian potential). The present study approaches the notions of Fan-hemicontinuity and (topological) pseudomonotonicity for operators. Pseudomonotone operators are presented in detail in [35],

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