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TOPOLOGICAL COMPLEXITY OF ORIENTED GRASSMANN MANIFOLDS

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Abstract. We study the topological complexity of the Grassmann manifolds $\widetilde{G}_{n,3}$ of oriented 3-dimensional vector subspaces in $\mathbb{R}^n$. By a result of Farber, for any field K, the topological complexity of a space X is greater than $\operatorname{zcl}_K(X)$, where $\operatorname{zcl}_K(X)$ is the K-zero-divisor cup-length of X. In this paper we examine $\operatorname{zcl}_{\mathbb{Z}_2}(\widetilde{G}_{n,3})$. Some lower and upper bounds for this invariant are obtained for all integers $n \geq 6$. For infinitely many of them the exact value of $\operatorname{zcl}_{\mathbb{Z}_2}(\widetilde{G}_{n,3})$ is computed, and in the rest of the cases these bounds differ by 1. We thus establish lower bounds for the topological complexity of Grassmannians $\widetilde{G}_{n,3}$.

1. Introduction

Let X be a path connected space and $\pi\colon X^I \to X \times X$ (X^I is the space of free paths in X) the fibration given by $\pi(\omega) = (\omega(0), \omega(1))$. The topological complexity of X, which we denote by $\mathrm{TC}(X)$, was defined by Farber in [7] as the Schwarz genus (see [20]) of this fibration, i.e. as the minimal integer m such that there exists an open cover $\{U_1, \dots, U_m\}$ of $X \times X$ with the property that π

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