Topological **M**ethods in **N**onlinear **A**nalysis
Volume 65, No. 1, 2025, 99–121
DOI: 10.12775/TMNA.2024.024

RESCALED-EXPANSIVE FLOWS: UNSTABLE SETS AND TOPOLOGICAL ENTROPY

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Abstract. In this work, we introduce and explore a rescaled theory of local stable and unstable sets for rescaled-expansive flows and its applications to topological entropy. We introduce a rescaled version of the local unstable sets and the unstable points. We find conditions for points of the phase space to exhibit non-trivial connected pieces of such unstable sets. We apply these results to the problem of proving positive topological entropy for rescaled-expansive flows with non-singular Lyapunov stable sets.

1. Introduction

The property of expansiveness introduced by R. Utz in 1950 is a landmark of the dynamical systems theory. Its great success is in part due to its proximity to the hyperbolic theory and its close relationship with many important topics of the dynamical systems theory, such as the stability theory and the entropy theory. Very soon, expansiveness was perceived as a source of complex dynamical behavior. Indeed, many expansive systems exhibit chaotic features. We refer the reader to [1] for a detailed exposition of the dynamical properties of expansive homeomorphisms.

The concept of expansive flow was introduced in [9] by R. Bowen and P. Walters to describe the behavior of Axiom A flows, but it is not appropriate to deal

2020 *Mathematics Subject Classification.* Primary: 37B05; Secondary: 37C70.
Key words and phrases. Rescaled-expansiveness; topological entropy; attractors.
A. Arbieto was partially supported by CAPES, CNPq, PRONEX-Dynamical Systems and FAPERJ E-26/201.181/2022 "Programa Cientista do Nosso Estado from Brazil".
E. Rego was partially supported by NSFC 12250710130 from China.