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INFINITELY MANY POSITIVE SOLUTIONS FOR A DOUBLE PHASE PROBLEMS INVOLVING THE DOUBLE PHASE OPERATOR

Chunbo Lian — Bin Ge — Qinghai Cao — Yu Zhang

Abstract. In this paper we study a double phase problem which involves the double phase operator, and the nonlinear term has an oscillatory behavior. By using variational methods and the theory of the Musielak–Orlicz–Sobolev space, we establish the existence of infinitely many solutions whose $W_0^{1,H}(\Omega)$-norms tend to zero (to infinity, respectively) whenever the nonlinearity oscillates at zero (at infinity, respectively).

1. Introduction

With the emergence of nonlinear problems in natural science and engineering, the Musielak–Orlicz–Sobolev space $W^{1,H}$ demonstrates their limitations in applications. A class of nonlinear problems involving the double phase operator, for example, is a new research field and reflects a new kind of physical phenomena. It possesses a solid background in physics and originates from the study

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