

## ON THE FRACTIONAL-IN-TIME KELLER–SEGEL MODEL VIA SONINE KERNELS

MASTERSON COSTA — CLAUDIO CUEVAS  
CLESSIUS SILVA — HERME SOTO

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**ABSTRACT.** In this paper, we study the existence and asymptotic behavior to a diffusion system which is non-local in time. As consequence of our theorems we deduce new results for the fractional-in-time Keller–Segel model. Our approach is intimately related with the Sonine kernels.

### 1. Introduction

Mathematical modeling of chemotaxis are of high scientific interest. Diffusion and chemotaxis are fundamental to the notion of bacteria, see e.g. [17] and references therein. The first modeling was introduced in the 1970s by Keller–Segel in [23] to describe the aggregation of slime mold amoebae due to an attractive chemical substance. The Keller–Segel system itself has been investigated for a longer period in the last decades, we refer the reader to [8], [9], [16], [18]–[21]. There is a serious mathematical interest in to develop fractional studies for the KS model. It is believed that this approach can be useful for modeling feeding strategies of a broad class of organisms, see e.g. [10]. Note that at the macroscopic level anomalous subdiffusion can be modeled by using a fractional temporal operator, see e.g. [2], [14], [26]–[28], [31], [32]. A fractional model for chemotaxis,

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