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PARTIAL MINIMIZATION OVER THE NEHARI SET AND APPLICATIONS TO ELLIPTIC EQUATIONS

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Abstract. We present a general scheme to find variationally characterized critical points of a functional $I\colon H \to \mathbb{R}$ on a Hilbert space H with hypothesis where the usual Nehari method is not directly applicable. These critical points arise as minima of I over a suitable subset of the associated Nehari set and are obtained with the aid of fibering methods. Moreover, we derive a comparison result with mountain pass critical values. The abstract results will be applied to classes of logarithmic Choquard and nonlinear Schrödinger equations.

1. Introduction

Let H be a Hilbert space with norm $\|\cdot\|$ of class $C^1(H\setminus\{0\};\mathbb{R})$ and $I\colon H \to \mathbb{R}$ a C^1-functional with $I(0) = 0$. The Nehari method is quite a powerful tool to find variationally characterized critical points for I. This method goes back to the work of Nehari in [14], [15] and is nowadays a standard tool in the calculus of variations, see e.g. [17], [11] and the references therein. To put our results in perspective, we briefly review the classical setting for functionals of the form $u \mapsto I(u) = \|u\|^2/2 - \psi(u)$, where the non-quadratic part ψ satisfies $\psi'(u) = o(\|u\|)$, $s \mapsto \psi'(su)/s$ is strictly increasing on $(0,\infty)$ for all $u \neq 0$ and $\psi(su)/s^2 \to \infty$ as $s \to \infty$ uniformly for u on weakly compact sets of $H \setminus \{0\}$.

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