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SECTIONAL CATEGORY OF MAPS RELATED TO FINITE SPACES

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Abstract. In this study, we compute some examples of sectional category $\mathrm{secat}(f)$ and sectional number $\mathrm{sec}(f)$ for continuous maps f related to finite spaces. Moreover, we introduce an invariant $\mathrm{secat}_k(f)$ for a map f between finite spaces using the k-th barycentric subdivision and show the equality $\mathrm{secat}_k(f) = \mathrm{secat}(\mathcal{B}(f))$ for sufficiently large k, where $\mathcal{B}(f)$ is the induced map on the associated polyhedra.

1. Introduction

The sectional category $\mathrm{secat}(f)$ of a continuous map $f\colon X \to Y$ between topological spaces X and Y is a numerical invariant originally introduced in [16]. This is defined as the smallest number n such that there exist $n+1$ open sets covering Y, where each open set admits a homotopy section of f. Several numerical invariants of topological spaces are expressed as the sectional category of special maps.

The *Lusternik–Schnirelmann* (LS) *category* $\mathrm{cat}(X)$ of a space X is the smallest number n such that there exist $n+1$ categorical open sets covering X (see [11]). Here, a subset $U \subset X$ is *categorical* if U is contractible in X; that is, the inclusion $U \hookrightarrow X$ is null-homotopic. The LS category $\mathrm{cat}(X)$ agrees with the sectional category $\mathrm{secat}(f)$ of a null-homotopic map to X. Furthermore, the

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