

NORMALIZED SOLUTIONS TO A CLASS OF CHOQUARD-TYPE EQUATIONS WITH POTENTIAL

LEI LONG — XIAOJING FENG

ABSTRACT. In this paper, we study the existence and nonexistence of solutions to the following Choquard-type equation

$$-\Delta u + (V + \lambda)u = (I_\alpha * F(u))f(u) \quad \text{in } \mathbb{R}^N,$$

having prescribed mass $\int_{\mathbb{R}^N} u^2 = a$, where $\lambda \in \mathbb{R}$ will arise as a Lagrange multiplier, $N \geq 3$, $\alpha \in (0, N)$, I_α is Riesz potential. Under suitable assumptions on the potential function V and the nonlinear term f , $a_0 \in [0, \infty)$ exists such that the above equation has a positive ground state normalized solution if $a \in (a_0, \infty)$ and one has no ground state normalized solution if $a \in (0, a_0)$ when $a_0 > 0$ by comparison arguments. Moreover, we obtain sufficient conditions for $a_0 = 0$.

1. Introduction

We consider the Choquard-type equation

$$(1.1) \quad -\Delta u + (V + \lambda)u = (I_\alpha * F(u))f(u) \quad \text{in } \mathbb{R}^N,$$

2020 *Mathematics Subject Classification*. Primary: 35D30, 35J20; Secondary: 35J50, 35Q55.

Key words and phrases. Choquard-type equations; normalized solutions; positive solutions.

The authors were supported in part by the National Natural Science Foundation of China (Grant Nos. 12026217, 12026218, 12271313, 12071266 and 12101376), Fundamental Research Program of Shanxi Province (202203021211300, 20210302124528, 202103021224013 and 202203021211309), and Shanxi Scholarship Council of China (2020-005).