

**ON THE CRITICAL EXPONENTS
FOR A FRACTIONAL DIFFUSION-WAVE EQUATION
WITH A NONLINEAR MEMORY TERM
IN A BOUNDED DOMAIN**

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ABSTRACT. In this paper, we prove sharp blow-up and global existence results for a time fractional diffusion-wave equation with a nonlinear memory term in a bounded domain, where the fractional derivative in time is taken in the sense of the Caputo type. Moreover, we also give a result for nonexistence of global solutions to a wave equation with a nonlinear memory term in a bounded domain. The proof of blow-up results is based on the eigenfunction method and the asymptotic properties of solutions for an ordinary fractional differential inequality.

1. Introduction

This paper is mainly concerned with the blow-up and global existence of solutions for the following time fractional diffusion-wave equation with a nonlinear memory term:

$$(1.1) \quad \begin{cases} {}_0D_t^\alpha u - \Delta u = {}_0I_t^\gamma(|u|^p), & (t, x) \in (0, T) \times \Omega, \\ u(t, x) = 0, & (t, x) \in (0, T) \times \partial\Omega, \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), & x \in \Omega, \end{cases}$$

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