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ON THE OPERATOR OF CENTER OF DISTANCES BETWEEN THE SPACES OF CLOSED SUBSETS OF THE REAL LINE

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Abstract. We study properties of an operator S which assigns to compact subsets of $[0,1]$ their centers of distances. We consider its continuity points and its upper semicontinuity points as well as orbits and fixed points of this operator. We also compute centers of distances of some classic sets. Using properties of operator S we show that the family of achievement sets is of the first category in the space of compact subsets of $[0,1]$.

1. Introduction

The notion of a center of distances is an interesting invariant of a metric space. Although this notion is new, it has applications to a much heavier studied subject-achievement sets of a sequences (called sets of subsums of series). For a given metric space X with a distance ρ *the center of distances* is defined as follows:

$$S(X) := \{\alpha : \forall\, x \in X\ \exists\, y \in X\ \rho(x,y) = \alpha\}.$$

W. Bielas, S. Plewik and M. Walczyńska, who introduced this notion in [8], show two interesting applications of center of distances of a perfect subset of the real line. We are going to refer to this in Section 3. This notion was also used in [6]

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