

NORMAL FORMS OF PARABOLIC LOGARITHMIC TRANSSERIES

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ABSTRACT. We give formal normal forms for parabolic logarithmic transseries $f = z + \dots$, with respect to parabolic logarithmic normalizations. Normalizations are given algorithmically, using fixed point theorems, as limits of Picard’s sequences in appropriate complete metric spaces, in contrast to transfinite *term-by-term* eliminations described in former works. Furthermore, we give the explicit formula for the residual coefficient in the normal form and show that, in the larger logarithmic class, we can even eliminate the residual term from the normal form.

1. Introduction

Transseries are generalizations of the standard power series. Shortly, transseries are formal sums of monomials obtained as formal products of powers, iterated exponentials and iterated logarithms (see [22]). Additionally, their sets of exponents (i.e., supports) are well-ordered sets, which is important for defining standard operations such as multiplication and composition of transseries. Transseries are nowadays an important tool for tackling problems in mathematics (see e.g. [5] and [6]) and physics (see [1]).

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