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MULTIPLE SOLUTIONS OF NONLINEAR NEUMANN INCLUSIONS

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Abstract. We prove new results on the existence of multiple solutions for elliptic inclusions with nonlinear boundary conditions of Neumann type. Our approach is topological and relies on the fixed point index for multivalued map.

1. Introduction

In this paper we discuss the existence of multiple solutions to the Neumann inclusion

$$\begin{cases} -\Delta u \in G(|x|,u) \quad \text{in } \Omega, \\ \dfrac{\partial u}{\partial r} = 0 \quad \text{on } |x|=R_0 \quad \text{and} \quad \dfrac{\partial u}{\partial r} = H[u] \quad \text{on } |x|=R_1, \end{cases} \tag{1.1}$$

where $\Omega = \{x \in \mathbb{R}^n : R_0 < |x| < R_1\}$ is the annulus, $0 < R_0 < R_1 < +\infty$, G is a multivalued map, H is a compact functional not necessarily linear and $\frac{\partial}{\partial r}$ denotes (as in [19]) differentiation in the radial direction $r = |x|$.

The approach that we want to use is topological and relies on multivalued fixed point index theory. In fact, under suitable transformations, radially symmetric solutions of a PDE can correspond to the solutions of an associated

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