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EXISTENCE AND MULTIPLICITY OF RADIALLY SYMMETRIC SOLUTIONS FOR NONLINEAR SCHRÖDINGER EQUATIONS

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Abstract. In this paper we study the following nonlinear Schrödinger equations in $\mathbb{R}^N$:

$$-\Delta u + V(x)u = g(u), \quad u \in H^1(\mathbb{R}^N),$$

where $N \geq 2$, $V \in C^1(\mathbb{R}^N, \mathbb{R})$ and $g \in C(\mathbb{R}, \mathbb{R})$. For a wide class of nonlinearities, which satisfy the Berestycki–Lions type condition, we show the existence and multiplicity of radially symmetric solutions. We use a new deformation argument under a new version of the Palais–Smale condition.

1. Introduction

In this paper, we study the existence and multiplicity of solutions of the following nonlinear Schrödinger equations in $\mathbb{R}^N$:

$$-\Delta u + V(x)u = g(u), \quad u \in H^1(\mathbb{R}^N), \tag{1.1}$$

where $N \geq 2$, $V \in C^1\big(\mathbb{R}^N, \mathbb{R}\big)$ is bounded and $g \in C(\mathbb{R}, \mathbb{R})$.

Since the pioneering work of Strauss [20] (see also [14], [15]), this problem is widely studied by many mathematicians. When $V(x) \equiv 0$ on $\mathbb{R}^N$, this problem is also called as nonlinear scalar field equation and in the celebrated work of Berestycki and Lions [5], [6] and Berestycki, Gallouët and Kavian [4], they gave almost optimal conditions for the existence of solutions. Namely via constraint

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