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PERIODIC SOLUTIONS WITH IRRATIONAL FREQUENCY FOR A CLASS OF SEMILINEAR WAVE EQUATIONS WITH VARIABLE COEFFICIENTS

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Abstract. This paper is devoted to the study of the existence of periodic solutions for a class of semilinear wave equations with variable coefficients. The forced vibrations of a nonhomogeneous string and propagation of seismic waves in nonisotropic media is governed by this mathematical model. When the frequency is a sufficiently large irrational number with bounded partial quotients, the existence of weak solutions is established. Then, under some suitable conditions, we improve the regularity of weak solutions. Our results can also be applied to the corresponding constant coefficients wave equation.

1. Introduction

In this paper, we construct periodic solutions to the following variable coefficients wave equation

$$\rho(x)y_{tt} - (\rho(x)y_x)_x + f(x,y) = g(t,x), \quad (t,x) \in \Omega := (0,T) \times (0,\pi), \tag{1.1}$$

with the periodic conditions

$$y(0,x) = y(T,x), \qquad y_t(0,x) = y_t(T,x), \tag{1.2}$$

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