Topological **M**ethods in **N**onlinear **A**nalysis
Volume 62, No. 2, 2023, 553–567
DOI: 10.12775/TMNA.2023.014

BALANCED CAPACITIES

Taras Radul

Abstract. We consider capacity (fuzzy measure, non-additive probability) on a compactum as a monotone cooperative normed game. Then it is natural to consider probability measures as elements of core of such game. We prove a topological version of the Bondareva–Shapley theorem that non-emptiness of the core is equivalent to balancedness of the capacity. We investigate categorical properties of balanced capacities and give characterizations of some fuzzy integrals of balanced capacities.

1. Introduction

The important solution concept of a cooperative games is the core notion [13]. The core consists of additive set functions which dominate the characteristic function of the game. The Bondareva–Shapley Theorem ([2] and [33]) provides a characterization of games on finite set with non-empty core. It states that the core is non-empty if and only if the game is balanced. The games on infinite sets were considered in [32], [16], [17], [23], [1], where the core consists as of additive set functions so and of σ-additive set functions.

We consider in this paper a cooperative game on a compactum. It seems natural to consider the core consisting of σ-additive regular normed measures (probability measures). We prove in Section 2 an analogue of Bondareva–Shapley theorem that non-emptiness of the core is equivalent to balancedness of the game.

Capacities (non-additive measures, fuzzy measures) were introduced by Choquet in [4] as a natural generalization of additive measures. Various non-additive

2020 *Mathematics Subject Classification.* 18C15, 28E10, 91A12, 54B30.
Key words and phrases. Balanced capacity; core of a cooperative game; fuzzy integral.