

## FIXED POINT THEOREMS IN PARTIALLY ORDERED TOPOLOGICAL SPACES WITH APPLICATIONS

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ABSTRACT. In this paper, we establish several new fixed point results in the framework of topological spaces endowed with a partial order. Special attention is paid to the case that the topology is induced by a metric. Our conclusions generalize many well-known results. Several examples and illustrative applications are provided to support the exposed results.

### 1. Introduction

In [7], R. Espinola and A. Wiśnicki proved the following fixed point theorem:

**THEOREM 1.1.** *Let  $X$  be a Hausdorff compact topological space with a partial order  $\preceq$  for which order intervals are closed and let  $T: X \rightarrow X$  be increasing. Assume that there exists  $c \in X$  such that  $c \preceq T(c)$ . Then  $T$  has a fixed point.*

The proof of this result is based on a combined use of topological methods and partial ordering techniques and relies on the Knaster–Tarski fixed point theorem.

In the same paper, the authors went on to show that a number of fixed point results related to monotone nonexpansive mappings are reducible to Theorem 1.1.

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