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# EXPONENTIAL ATTRACTOR FOR THE CAHN–HILLIARD–OONO EQUATION IN $\mathbb{R}^N$

Jan W. Cholewa — Radosław Czaja

Abstract. We consider the Cahn–Hilliard–Oono equation in the whole of $\mathbb{R}^N$, $N \leq 3$. We prove the existence of an exponential attractor in $H^1(\mathbb{R}^N)$, which contains a global attractor. We also estimate from above fractal dimension of the attractors.

## 1. Introduction

The Cahn–Hilliard equation is well known in material sciences involving phase separation processes, see e.g. [8], [52], [46], [43] and references therein. Several variations are also of broader interest; multi-component alloy models [29], [11], [36], models with viscosity or inertial terms [28], [45], [40], [9], [31], the Cahn–Hilliard–Cook equations [3], [4], [51], and the Cahn–Hilliard–Oono model [41], [30], which in turn involves hyperbolic relaxation models [49], [50] and Navier–Stokes equations [44].

In contrast with the case of bounded domains much less references deal with the Cahn–Hilliard type models in unbounded domains; see [7], [6], [35], [5], [38], [23], [22], [14], [15], [48], [49], [50], [55], [54] in chronological order. Specifically

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