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CONCENTRATING SOLUTIONS FOR A BIHARMONIC PROBLEM WITH SUPERCRITICAL GROWTH

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Abstract. In this paper we consider the following supercritical biharmonic problem:

$$\begin{cases} \Delta^2 u = K(x) u^{p+\varepsilon} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $K(x) \in C^3(\overline{\Omega})$ is a nonnegative function, $p = (N+4)/(N-4)$, $\varepsilon > 0$, Ω is a smooth bounded domain in $\mathbb{R}^N$, $N \geq 6$. We show that, for ε small enough, there exists a family of concentrating solutions under certain assumptions on the critical points of the function $K(x)$.

1. Introduction

In this paper, we study the following semilinear biharmonic equation under the Navier boundary condition

$$\begin{cases} \Delta^2 u = K(x) u^{p+\varepsilon} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

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