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ON A CLASS OF WEIGHTED ANISOTROPIC p-LAPLACE EQUATION WITH SINGULAR NONLINEARITY

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Abstract. We consider a class of singular weighted anisotropic p-Laplace equations. We provide sufficient condition on the weight function that may vanish or blow up near the origin to ensure the existence of at least one weak solution in the purely singular case and at least two different weak solutions in the purturbed singular case.

1. Introduction

In this article, we establish existence of weak solutions for the following class of weighted anisotropic singular problems

$$-F_{p,w}u = g(x,u) \quad \text{in } \Omega, \qquad u > 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega, \tag{1.1}$$

where $1 < p < \infty$, $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain with $N \geq 2$. Here

$$F_{p,w}u := \operatorname{div}(w(x)F(\nabla u)^{p-1}\nabla_\xi F(\nabla u))$$

is the weighted anisotropic p-Laplace operator, where $F\colon \mathbb{R}^N \to [0,\infty)$ is the Finsler–Minkowski norm, that is

(H0) $F(x) \geq 0$, for every $x \in \mathbb{R}^N$.
(H1) $F(x) = 0$, if and only if $x = 0$.

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