

CONVERGENCE AND WELL-POSEDNESS PROPERTIES OF UNIFORMLY LOCALLY CONTRACTIVE MAPPINGS

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ABSTRACT. In a 1961 paper by E. Rakotch it was shown that a uniformly locally contractive mapping has a fixed point. In the present paper we show that for such a mapping, the fixed point problem is well posed and that inexact iterates of such a mapping converge to its unique fixed point, uniformly on bounded sets. Using the porosity notion, we also show that most uniformly locally nonexpansive mappings are, in fact, uniformly locally contractive.

1. Introduction

For almost sixty years now, there has been a lot of research activity regarding the fixed point theory of nonexpansive (that is, 1-Lipschitz) mappings. See, for example, [2]–[4], [6]–[9], [12]–[17], [19], [21]–[23], [25], [26], [29], [30] and references cited therein. This activity stems from Banach’s classical theorem [1] concerning the existence of a unique fixed point for a strict contraction. It also concerns the convergence of (inexact) iterates of a nonexpansive mapping to one of its fixed points. Since that seminal result, many developments have taken place in this area including, in particular, studies of feasibility and common

2020 *Mathematics Subject Classification*. Primary: 47H09, 47H10; Secondary: 54E50.

Key words and phrases. Complete metric space; fixed point; inexact iterate; nonexpansive mapping.

The first author was partially supported by the Israel Science Foundation (Grant No. 820/17), by the Fund for the Promotion of Research at the Technion and by the Technion General Research Fund.