

## LOWER SEMICONTINUITY OF KIRCHHOFF-TYPE ENERGY FUNCTIONALS AND SPECTRAL GAPS ON (SUB)RIEMANNIAN MANIFOLDS

CSABA FARKAS — SÁNDOR KAJÁNTÓ — CSABA VARGA

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ABSTRACT. In this paper we characterize the sequentially weakly lower semicontinuity of the parameter-dependent energy functional associated with the critical Kirchhoff problem in context of (sub)Riemannian manifolds. We also present some spectral gap and convexity results.

### 1. Introduction

Let  $\Omega \subset \mathbb{R}^n$  be an open bounded domain and consider the generalized eigenvalue problem

$$(\mathcal{P}_{a,b,\lambda}(\mathbb{R}^n)) \quad \begin{cases} -\mathcal{K}u = \lambda u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where  $\lambda$  is a positive parameter,  $u: \bar{\Omega} \rightarrow \mathbb{R}$  is the unknown function,

$$\mathcal{K}u := \left( a + b \int_{\Omega} |\nabla u|^2 dx \right) \Delta u + |u|^{2^*-2}u$$

is the so-called critical Kirchhoff-type operator and  $2^* = 2n/(n-2)$  is the critical Sobolev exponent, with  $n \geq 3$ .

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