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EXISTENCE OF NONTRIVIAL SOLUTIONS TO SCHRÖDINGER SYSTEMS WITH LINEAR AND NONLINEAR COUPLINGS VIA MORSE THEORY

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Abstract. In this paper, we use Morse theory to study existence of nontrivial solutions to the following Schrödinger system with linear and nonlinear couplings which arises from Bose–Einstein condensates:

$$\begin{cases} -\Delta u + \lambda_1 u + \kappa v = \mu_1 u^3 + \beta u v^2 & \text{in } \Omega, \\ -\Delta v + \lambda_2 v + \kappa u = \mu_2 v^3 + \beta v u^2 & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain in $\mathbb{R}^N (N = 2, 3)$, $\lambda_1, \lambda_2, \mu_1, \mu_2 \in \mathbb{R} \setminus \{0\}$, $\beta, \kappa \in \mathbb{R}$. In two cases of $\kappa = 0$ and $\kappa \neq 0$, by transferring an eigenvalue problem into an algebraic problem, we compute the Morse index and critical groups of the trivial solution. Furthermore, even when the trivial solution is degenerate, we show a local linking structure of energy functional at zero within a suitable parameter range and then get critical groups of the trivial solution. As an application, we use Morse theory to get an existence theorem on existence of nontrivial solutions under some conditions.

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