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MULTIPLICITY OF POSITIVE SOLUTIONS FOR A KIRCHHOFF TYPE PROBLEM WITHOUT ASYMPTOTIC CONDITIONS

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Abstract. In this paper, we are concerned with the multiplicity of positive solutions for the following Kirchhoff type problem

$$\begin{cases} -\left(\varepsilon^2 a + \varepsilon b \displaystyle\int_{\mathbb{R}^3} |\nabla u|^2\,dx\right)\Delta u + u = Q(x)|u|^{p-2}u, & x \in \mathbb{R}^3, \\ u \in H^1(\mathbb{R}^3), \quad u > 0, & x \in \mathbb{R}^3, \end{cases}$$

where $\varepsilon > 0$ is a small parameter, $a, b > 0$ are constants, $4 < p < 6$, Q is a nonnegative continuous potential and does not satisfy any asymptotic condition. Combining Nehari manifold and concentration compactness principle, we study how the shape of the graph of $Q(x)$ affects the number of positive solutions.

1. Introduction and main results

In this paper, we are interested in the multiplicity of positive solutions to a class of semilinear Kirchhoff type problem:

$$\begin{cases} -\left(\varepsilon^2 a + \varepsilon b \displaystyle\int_{\mathbb{R}^3} |\nabla u|^2\,dx\right)\Delta u + u = Q(x)|u|^{p-2}u, & x \in \mathbb{R}^3, \\ u \in H^1(\mathbb{R}^3), \quad u > 0, & x \in \mathbb{R}^3, \end{cases} \tag{1.1}$$

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