

A FAMILY OF DISTAL FUNCTIONS AND MULTIPLIERS FOR STRICT ERGODICITY

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ABSTRACT. We give two proofs to an old result of E. Salehi, showing that the Weyl subalgebra \mathcal{W} of $\ell^\infty(\mathbb{Z})$ is a proper subalgebra of \mathcal{D} , the algebra of distal functions. We also show that the family \mathcal{S}^d of strictly ergodic functions in \mathcal{D} does not form an algebra and hence in particular does not coincide with \mathcal{W} . We then use similar constructions to show that a function which is a multiplier for strict ergodicity, either within \mathcal{D} or in general, is necessarily a constant. An example of a metric, strictly ergodic, distal flow is constructed which admits a non-strictly ergodic 2-fold minimal self-joining. It then follows that the enveloping group of this flow is not strictly ergodic (as a T -flow). Finally we show that the distal, strictly ergodic Heisenberg nil-flow is relatively disjoint over its largest equicontinuous factor from the universal Weyl flow $|\mathcal{W}|$.

1. Introduction

This work was originally written in the late 1980s. It was then circulated as a preprint among a few colleagues (see e.g. MathSciNet review: 986700) but for some reason was never published. In a recent conversation, Hillel Furstenberg asked whether there exists a subalgebra of $\ell^\infty(\mathbb{Z})$ which is contained in the subalgebra of distal functions, consists of strictly ergodic functions, and is “universal” in some sense. In a way Section 3 of this work gives a negative answer to Furstenberg’s question. So I decided to resurrect this old work. The new version differs from the original one mostly in the addition of a proof of Theorem 3.5,

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