

EXISTENCE OF SOLUTIONS FOR THE BREZIS–NIRENBERG PROBLEM

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ABSTRACT. We are concerned with of existence of solutions to the semilinear elliptic problem

$$\begin{cases} -\Delta u = \lambda_k u + u^3 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in a bounded domain $\Omega \subset \mathbb{R}^4$. Here λ_k is an eigenvalue of the $-\Delta$ in $H_0^1(\Omega)$. We prove that this problem has a nontrivial solution.

1. Introduction

Let Ω be a given bounded domain in \mathbb{R}^N , $N \geq 4$, with smooth boundary $\partial\Omega$. We consider the semilinear elliptic problem

$$(P) \quad \begin{cases} -\Delta u = \lambda u + |u|^{2^*-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $2^* = 2N/(N-2)$ and λ is a real parameter. Denote by $0 < \lambda_1 < \lambda_2 \leq \dots \leq \lambda_j \leq \dots$ the eigenvalues of $-\Delta$ in $H_0^1(\Omega)$.

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