

## LOCAL MORREY ESTIMATE IN MUSIELAK–ORLICZ–SOBOLEV SPACE

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ABSTRACT. Under appropriate assumptions on the  $N(\Omega)$ -function, locally uniform Morrey estimate is presented in the Musielak–Orlicz–Sobolev space. The assumptions include a new increasing condition on the  $x$ -derivative of the Young complementary function of the  $N(\Omega)$ -function. The conclusion applies to several important nonlinear examples frequently appeared in mathematical literature.

### 1. Introduction

Vast mathematical literature describes various aspects of partial differential equations related to the elliptic type operators including variable exponent, weighted, convex and double phase cases. The examples of the mentioned cases can be found in [13], [12], [23], [24], [1], [8], [7] and the references therein. Musielak–Orlicz–Sobolev spaces give an abstract framework of functional analysis to cover all of the above mentioned cases.

Applications in mathematical models dealt in the framework of Musielak–Orlicz settings start from Ball’s classical paper [4] on elasticity, investigated in the recent paper [5], [20] for the model about thermo-visco-plasticity type. We refer to [15]–[18], [29], [30] for the developments arising from the non-Newtonian

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