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CRITICAL KIRCHHOFF-TYPE EQUATION WITH SINGULAR POTENTIAL

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Abstract. In this paper, we deal with the following Kirchhoff-type equation:

$$-\left(1+\int_{\mathbb{R}^3}|\nabla u|^2\,dx\right)\Delta u+\frac{A}{|x|^\alpha}\,u=f(u),\quad x\in\mathbb{R}^3,$$

where $A>0$ is a real parameter and $\alpha\in(0,1)\cup(4/3,2)$. Remark that $f(u)=|u|^{2^*_\alpha-2}u+\lambda|u|^{q-2}u+|u|^4u$, where $\lambda>0$, $q\in(2^*_\alpha,6)$, $2^*_\alpha=2+4\alpha/(4-\alpha)$ is the embedding bottom index, and 6 is the embedding top index and Sobolev critical exponent. We point out that the nonlinearity f is the almost "optimal" choice. First, for $\alpha\in(4/3,2)$, applying the generalized version of Lions-type theorem and the Nehari manifold, we show the existence of nonnegative Nehari-type ground sate solution for above equation. Second, for $\alpha\in(0,1)$, using the generalized version of Lions-type theorem and the Pohožaev manifold, we establish the existence of nonnegative Pohožaev-type ground state solution for above equation. Based on our new generalized version of Lions-type theorem, our works extend the results in Li-Su [Z. Angew. Math. Phys. **66** (2015)].

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