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α-(h,e)-CONVEX OPERATORS AND APPLICATIONS FOR RIEMANN–LIOUVILLE FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we consider a class of α-(h,e)-convex operators defined in set $P_{h,e}$ and applications with $\alpha > 1$. Without assuming the operator to be completely continuous or compact, by employing cone theory and monotone iterative technique, we not only obtain the existence and uniqueness of fixed point of α-(h,e)-convex operators, but also construct two monotone iterative sequences to approximate the unique fixed point. At last, we investigate the existence-uniqueness of a nontrivial solution for Riemann-Liouville fractional differential equations integral boundary value problems by employing α-(h,e)-convex operators fixed point theorem.

1. Introduction

It is well known that concave(convex) operators defined on a cone play an important role in the fixed point theory and functional analysis. It is worthy to mention the recent papers [18], [7], [10], [28], [20], [19], [11], the authors studied various fixed point theorems, such as generalized concave operator, α-concave(convex) operator, t-$\alpha(t)$ mix monotone operator, e-concave-convex operator, t-$\alpha(t,u,v)$ mixed monotone operator, τ-φ-convex operator and so on, and

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