

## ON THE RELATIVE CATEGORY IN THE BRAKE ORBITS PROBLEM

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*Dedicated to the memory of Edward Fadell and Sufian Husseini*

ABSTRACT. In this paper we show how the notion of the Lusternik–Schnirelmann relative category can be used to study a multiplicity problem for brake orbits in a potential well which is homeomorphic to the  $N$ -dimensional unit disk. The estimate of the relative category of the set of chords with endpoints on the  $(N - 1)$ -unit sphere was shown to the third author by Fadell and Husseini while he was visiting the University of Wisconsin at Madison.

### 1. Introduction

Algebraic topology plays a fundamental role in many areas of mathematics. In the specific case of calculus of variations, algebraic topology provides a number of topological invariants that can be used to give lower estimates of the number of solutions to variational problems. Typically, the definition of such invariants uses appropriate variants of (relative) homology/cohomology theory or, as in the case of the celebrated Lusternik–Schnirelman category which has a lower bound in terms of the cuplength, they can be estimated using homological/cohomological techniques. Recall that the *Lusternik–Schnirelman category* of a topological space  $X$ , denoted by  $\text{cat}(X)$ , is the minimal integer  $k \in \mathbb{N} \cup \{+\infty\}$  such that  $X$  admits a covering formed by  $k$  closed contractible subsets. The reader will

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2020 *Mathematics Subject Classification*. Primary: 58E05; Secondary: 58E10, 37J46.

*Key words and phrases*. Lusternik–Schnirelmann category; variational inequalities, brake orbits.