

ON RADIAL SOLUTION FOR SOME ELLIPTIC EQUATIONS INVOLVING OPERATORS WITH UNBOUNDED COEFFICIENTS IN EXTERIOR DOMAINS

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ABSTRACT. We study existence and multiplicity of radial solutions for some quasilinear elliptic problems involving the operator $L_N = \Delta - x \cdot \nabla$ on $\mathbb{R}^N \setminus B_1$, where Δ is the Laplacian, $x \cdot \nabla$ is an unbounded drift term, $N \geq 3$ and B_1 is the unit ball centered at the origin. We consider: (i) Eigenvalue problems, and (ii) Problems involving a nonlinearity of concave and convex type. On the first class of problems we get a compact embedding result, whereas on the second, we address the well-known question of Ambrosetti, Brezis and Cerami from 1993 concerning the existence of two positive solutions for some problems involving the supercritical Sobolev exponent in symmetric domains for the Laplacian. Specifically, we provide a new approach of answering the ABC-question for elliptic problems with unbounded coefficients in exterior domains and we find asymptotic properties of the radial solutions. Furthermore, we study the limit case, namely when nonlinearity involves a sublinear term and a linear term. As far as we know, this is the first work that deals with such a case, even for the Laplacian. In our approach, we use both topological and variational arguments.

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