Topological **M**ethods in **N**onlinear **A**nalysis
Volume 60, No. 2, 2022, 653–672
DOI: 10.12775/TMNA.2022.022

TIME-DEPENDENT GLOBAL ATTRACTORS FOR THE STRONGLY DAMPED WAVE EQUATIONS WITH LOWER REGULAR FORCING TERM

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Abstract. In this paper, based on a new theoretical framework of time-dependent global attractors (Conti, Pata and Temam [8]), we consider the strongly damped wave equations $\varepsilon(t)u_{tt} - \Delta u_t - \Delta u + f(u) = g(x)$ and establish the existence of attractors in $\mathcal{H}_t = H_0^1(\Omega) \times L^2(\Omega)$ and $\mathcal{V}_t = H_0^1(\Omega) \times H_0^1(\Omega)$, respectively.

1. Introduction

We examine the existence of attractors for the following strongly damped wave equations:

$$\begin{cases} \varepsilon(t)u_{tt} - \Delta u_t - \Delta u + f(u) = g(x) & \text{in } \Omega \times (\tau, \infty), \\ u(x,t) = 0 & \text{on } \partial\Omega \times (\tau, \infty), \\ u(x,\tau) = u_\tau,\ u_t(x,\tau) = u'_\tau & \text{for } x \in \Omega, \end{cases} \tag{1.1}$$

in a bounded domain $\Omega \subset \mathbb{R}^3$ with smooth boundary. Here $\tau \in \mathbb{R}$ is the initial time, the external forcing $g(\,\cdot\,) \in H^{-1}(\Omega)$, $\varepsilon(\,\cdot\,) \in C^1(\mathbb{R})$ is a decreasing bounded

2020 *Mathematics Subject Classification.* Primary: 35B40, 35B41; Secondary: 35L05.
Key words and phrases. Strongly damped wave equations; critical exponential growth; time-dependent global attractors.
This work was supported in part by the NSFC (Grants No. 11901194), the Hunan Province Natural Science Foundation of China (Grants No. 2022JJ30417) and the China Postdoctoral Science Foundation (Grants No. 2021M692198).